CHAPTER 34 Maxwell's Equations and Electromagnetic Waves

Answers to Understanding the Concepts Questions

1. No. Polarization is a unique property of transverse waves, such as electromagnetic waves (including light). Sound waves are longitudinal and therefore they cannot be polarized.

2. No force is associated with uniformly moving electric charges, so that no work is done, and no energy is expended. When charges accelerate, however, then forces must act, work will be done, and energy will be expended.

3. The ionosphere has an abundant supply of free charges, which can oscillate at the same frequency of the incident radiowave, and in doing so they act as new sources of radiowave of the same frequency. Therefore the ionosphere reflects radiowaves well. This is somewhat like the behavior of a metal, which is a good reflector of electromagnetic waves thanks to its free electrons, and that is the reason why it is shiny (as it reflects visible light).

4. Bulb B1 is totally dark because no current runs through the wire, as the electric field vector in the EM wave is perpendicular to it. To make B1 shine brightly, just turn the wire connected to it by 90 degrees so the electric field vector points along the wire. Make sure the wire remains in the pane of the page, as this is the plane in which the electric field vector lies.

5. If the electric field in an electromagnetic wave has a definite orientation, so does the magnetic field, because the wave is transverse with $\vec{E}\cdot\vec{B}=0$. Thus the magnetic field characterizes the polarization just as well as the electric field. The electric field is used as the measure of the polarization because its effect on matter is more direct.

6. Yes. Standing electromagnetic waves can be produced when a traveling EM wave is reflected off of a surface and bounces back. The superposition of two EM waves traveling in opposite directions, namely the incident wave and the reflected one, results in a standing EM wave, much like a mechanical standing wave on a string. In general, when incident upon a boundary between two media, an EM wave is partly reflected back into the first medium and partly transmitted into, or absorbed by, the second one. The electromagnetic characteristics of the two media, such as the values of their dielectric constant and magnetic susceptibility, determine the ratio of reflection versus transmission/absorption. This in turn controls the value of the EM waves on the boundary.

7. No. Regardless of the arrangement, as the EM wave passes through the last linear polarizer it must be polarized along the axis of that polarizer.

8. To answer this question, we might want to think about the effects of electric charge. The very existence of magnetic monopoles will not change the nature of electromagnetic waves any more than the very existence of electric charge does. They would merely supply new ways to generate those waves. How about the presence of magnetic charge in free space where electromagnetic waves propagate? The fields of an electromagnetic wave cause free electric charge in its path to accelerate; these charges will accordingly reradiate. One effect is to change the effective propagation speed, others have to do with the direction of propagation. We expect the presence of magnetic monopoles to have similar effects.

9. The optical pressure from the Sun is not very significant. To achieve reasonable acceleration with it, the solar sails must be made of relatively "light" materials (i.e., with lower densities), and the area of the sail facing the Sun must be significant to increase the total optical force exerted on the sail. To maximize the optical pressure, the sails should be highly reflective and face the sunlight perpendicularly.

10. Yes. What is important in the ejection of mass is that the mass carries momentum. The principle of the conservation of momentum ensures the propulsion of the rocket. We have seen in this chapter that light also carries momentum, so that it could fulfill the same role as the ejected mass. Of course, whether the quantitative result is the same is another question entirely.

11. Place a linear polarizer facing the incoming light. Rotate the polarizer to check if the intensity of the transmitted light changes. If it does not then the light is not polarized. If it varies but does not reach zero at any point then the light is partially polarized. If no light is transmitted when the polarizer is set at a certain orientation then the light is linearly polarized, with the direction of polarization perpendicular to the axis of the polarizer.

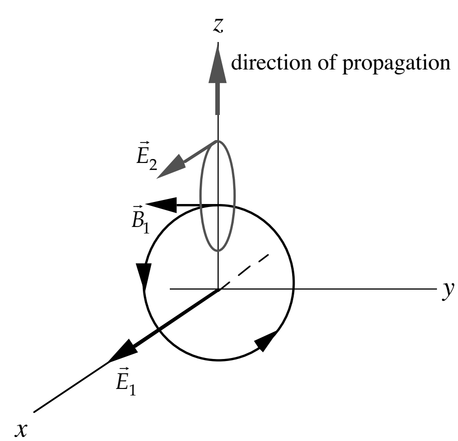
12. At least two polarizers are needed. (A single one polarized along the y-axis permits no light to go through as the incoming wave is polarized along the x-axis). We can place the axis of the first polarizer at 45 degrees from both the x- and y-axes, so the amplitude of the wave is reduced by a factor of $\cos 45° = 1/\sqrt{2}$ after p[assign through it. Now, place the axis of the second polarizer on the y-axis, so the amplitude of the wave is again reduced by a factor of $\cos 45° = 1/\sqrt{2}$. Overall, the amplitude of the transmitted wave is $(\cos 45°)^2 = 1/2$ of the incident one and the intensity is reduced by a factor of $(1/2)^2 = 0.25$. With three polarizers it is possible to reduce the loss in intensity. These polarizers can be positioned with their axes at 30 from each other. The wave amplitude is now reduced by $(\cos 30°)^3 = 0.65$, and the intensity drops by a factor of $(0.65)^2 = 0.42$ rather than 0.25.

13. When the lightbulb glows the brightest the electric field of the EM wave is parallel to the rod that supplies current to the bulb. If the rod is then rotated by 90° then the light goes out, as the electric field is now perpendicular to the rod and cannot support a current that flows along the rod to light up the lightbulb.

14. There is really nothing to check, because there is little difference between electromagnetic waves and Faraday induction. Faraday's law states that a time-dependent magnetic field gives rise to an electric field, and as we saw in this chapter, this interdependence is what gives rise to the electromagnetic waves. Of course the Maxwell generalization of Ampere's law is also crucial.

15. The electrical power consumed by the lightbulb (100 W) turns into electromagnetic radiation energy, some in the visible light range (so the bulb glows), some in the infrared range (so it is hot to the touch). Eventually, all this radiation energy, regardless of the frequency, is absorbed by the room and is used to heat up the room.

16. Consider an electromagnetic wave produced by an arbitrary source, which does not necessarily exhibit any form of geometrical symmetry. Suppose that, at one instant, the electric field in a small local region is $\vec{E}_1$, which points in the positive x-direction. For the sake of definitiveness let's assume that the magnitude of $\vec{E}_1$ is increasing at this moment. According to the theory of displacement current a corresponding magnetic field is generated ($\vec{B}_1$), which loops around (and is perpendicular to) $\vec{E}_1$. Consider a point on the loop that intersects with the z-axis. The direction of $\vec{B}_1$ at that point is



in the negative y-axis, as shown. Since $\vec{E}_1$ is increasing, so is $\vec{B}_1$. According to the Faraday's law of induction, an induced electric field, $\vec{E}_2$, must be produced in the xz plane (perpendicular to $\vec{B}_1$). All told, the original electric field $\vec{E}_1$ has propagated to a new location further up the z-axis, producing a secondary field $\vec{E}_2$. The direction of propagation of the electromagnetic wave in this case is the positive z-direction, which is perpendicular to $\vec{E}_1$ (which is in the x-direction). Now, if you start with a magnetic field in place of $\vec{E}_1$ and follow a similar analysis, you can easily see that the propagation direction of the wave is also perpendicular to the magnetic filed. Thus the electromagnetic wave is transverse.

17. Let's suppose that an electromagnetic wave, propagating in the $+z$-direction, is linearly polarized, with the electric field initially increasing in the $+x$-direction; that means that the magnetic field will be increasing in the $+y$-direction. Let's also put the positive (negative) charge of the dipole at positive (negative) x to start. The forces on the dipole charges accelerate the positive charge to the right and the negative charge to the left; that is, they tend to spread the charges in the dipole. Once these charges move, the magnetic field exerts forces on them. The force on the positive charge is to the positive z-direction, while the force on the negative charge is also to the positive z-direction. There is no net torque, and a linearly polarized electromagnetic wave carries no angular momentum. However, when we consider a circularly polarized wave, in which a superposition of two linearly polarized waves makes a wave consisting of electric and magnetic fields remaining constant in magnitude while rotating in the xy-plane, an analysis like the one we described above shows that the wave carries angular momentum with a z-component.

18. Yes. The three vectors, $\vec{E}$, $\vec{B}$ and $\hat{k}$ ($\hat{k}$ is along the direction of propagation) satisfy the right-hand-rule: $\vec{E} \times \vec{B}$ should be in the direction of $\hat{k}$.

19. A reflective sail is better. Ideally, a totally reflective sail changes the momentum of the incident light beam by twice as much as a totally black one, since it reverses the momentum of the incident beam rather than absorbing it. So the optical pressure exerted on a reflective sail is up to twice as much as that on a totally black one.

20. Yes, we will indeed pick up less of a signal from a sending antenna if there is a receiving antenna between us and the sending antenna, at least if the receiving antenna expends energy in any way, such as driving speakers. The conservation of energy ensures us that this must be true.

21. The amplitude of the magnetic field in an electromagnetic wave is equal to that of the electric field divided by the speed of light. So for a charged particle in the medium whose velocity component perpendicular to the B-field is v, the electric force exerted on it by the electromagnetic wave is much, much greater than the magnetic force: $F_E / F_B = qE_{\max} / qvB_{\max} = qE_{\max} / [qv(E_{\max} / c)] = c / v >> 1$; hence the electric field dominates the behavior of the wave in the medium.

22. The signal from a distant radio station, if powerful enough, can bounce off the ionosphere to be received by your radio — refer to the discussion on Question 3.

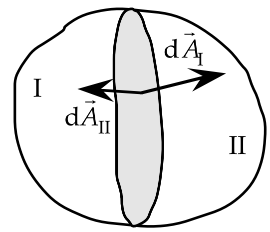
Solutions to Problems

1. For Gauss' law for electric fields, we have
   $[E][A] = [Q][\varepsilon_0]^{-1};$
   $[ML/QT^2][L]^2 = [Q][Q^2T^2/ML^3]^{-1};$
   $[ML^3/QT^2] = [ML^3/QT^2].$
   For Gauss' law for magnetic fields, we have
   $[B][A] = 0;$
   $[M/QT][L]^2 = 0;$
   $[ML^2/QT] = 0.$
   For the generalized Ampere's law, we have
   $[B][s] = [\mu_0][I] + [\mu_0\varepsilon_0][E][A][t]^{-1};$
   $[M/QT][L] = [ML/Q^2][Q/T] + [T/L]^2\,[ML/QT^2][L]^2\,[T]^{-1};$
   $[ML/QT] = [ML/QT] + [ML/QT].$
   For Faraday's law, we have
   $[E][s] = [B][A]\,[t]^{-1};$
   $[ML/QT^2][L] = [M/QT][L]^2\,[T]^{-1};$
   $[ML^2/QT^2] = [ML^2/QT^2].$

2. We will use M for the magnetic monopole, to distinguish it from the dimension M.
   From the analogy to Gauss' law for electric fields, we have
   $\oint\!\!\oint \vec{B}\cdot \mathrm{d}\vec{A} = \mu_0 \mathsf{M}.$
   We find the dimensions of M from
   $[B]\,[A] = [\mu_0]\,[\mathsf{M}];$
   $[M/QT][L]^2 = [ML/Q^2][\mathsf{M}]$, which gives
   $[\mathsf{M}] = [QL/T]$, with SI units of $\boxed{\mathrm{A\cdot m}}$.

3. From the analogy to Ampere's law, we have
   $$\oint \vec{E}\cdot \mathrm{d}\vec{s} = \mu_0 \frac{\mathrm{d}\mathsf{M}}{\mathrm{d}t} - \frac{\mathrm{d}}{\mathrm{d}t}\iint_S \vec{B}\cdot \mathrm{d}\vec{A}.$$
   The $\mathrm{d}\mathsf{M}/\mathrm{d}t$ term corresponds to an electric field created by the "current" of magnetic monopoles.

4. We can write the closed surface integral for each of the subregions as the sum of the integral over the part of the whole surface and the integral over the common surface:



$$\oiint \vec{E}\cdot d\vec{A}_{\mathrm{I}} = \iint_{\text{whole}} \vec{E}\cdot d\vec{A}_{\mathrm{I}} + \iint_{\text{common}} \vec{E}\cdot d\vec{A}_{\mathrm{I}};$$
$$\oiint \vec{E}\cdot d\vec{A}_{\mathrm{II}} = \iint_{\text{whole}} \vec{E}\cdot d\vec{A}_{\mathrm{II}} + \iint_{\text{common}} \vec{E}\cdot d\vec{A}_{\mathrm{II}}.$$

Because $d\vec{A}$ points outward from each enclosed volume, over the common surface $d\vec{A}_{\mathrm{I}} = -\,d\vec{A}_{\mathrm{II}}$, so we have

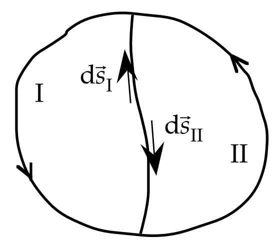
$$\iint_{\text{common}} \vec{E}\cdot d\vec{A}_{\mathrm{I}} = -\iint_{\text{common}} \vec{E}\cdot d\vec{A}_{\mathrm{II}}, \ \text{ and } \iint_{\text{common}} \vec{B}\cdot d\vec{A}_{\mathrm{I}} = -\iint_{\text{common}} \vec{B}\cdot d\vec{A}_{\mathrm{II}}.$$

If we add the closed surface integrals for the two subregions, we get

$$\oiint \vec{E}\cdot d\vec{A}_{\mathrm{I}} + \oiint \vec{E}\cdot d\vec{A}_{\mathrm{II}} =$$
$$\iint_{\text{whole}} \vec{E}\cdot d\vec{A}_{\mathrm{I}} + \iint_{\text{common}} \vec{E}\cdot d\vec{A}_{\mathrm{I}} + \iint_{\text{whole}} \vec{E}\cdot d\vec{A}_{\mathrm{II}} + \iint_{\text{common}} \vec{E}\cdot d\vec{A}_{\mathrm{II}} = \oiint_{\text{whole}} \vec{E}\cdot d\vec{A};$$
$$\oiint \vec{B}\cdot d\vec{A}_{\mathrm{I}} + \oiint \vec{B}\cdot d\vec{A}_{\mathrm{II}} =$$
$$\iint_{\text{whole}} \vec{B}\cdot d\vec{A}_{\mathrm{I}} + \iint_{\text{common}} \vec{B}\cdot d\vec{A}_{\mathrm{I}} + \iint_{\text{whole}} \vec{B}\cdot d\vec{A}_{\mathrm{II}} + \iint_{\text{common}} \vec{B}\cdot d\vec{A}_{\mathrm{II}} = \oiint_{\text{whole}} \vec{B}\cdot d\vec{A};$$

The charge is additive, $Q = Q_{\mathrm{I}} + Q_{\mathrm{II}}$, so for every term of Maxwell's first and second equations the sum of the terms for the two subregions is the term for the whole region. Thus Maxwell's first and second equations are valid for the whole region.

If we choose a path in each region which has a common segment, we can write the closed line integral for each of the subregions as the sum of the integral over the part on the whole surface and the integral over the common segment:



$$\oint \vec{E}\cdot d\vec{s}_{\mathrm{I}} = \int_{\text{whole}} \vec{E}\cdot d\vec{s}_{\mathrm{I}} + \int_{\text{common}} \vec{E}\cdot d\vec{s}_{\mathrm{I}};$$
$$\oint \vec{E}\cdot d\vec{s}_{\mathrm{II}} = \int_{\text{whole}} \vec{E}\cdot d\vec{s}_{\mathrm{II}} + \int_{\text{common}} \vec{E}\cdot d\vec{s}_{\mathrm{II}};$$

We take the same circulation for the path in each subregion. Because $d\vec{s}$ points in opposite directions over the common segment, $d\vec{s}_{\mathrm{I}} = -\,d\vec{s}_{\mathrm{II}}$, so we have

$$\int_{\text{common}} \vec{E}\cdot d\vec{s}_{\mathrm{I}} = -\int_{\text{common}} \vec{E}\cdot d\vec{s}_{\mathrm{II}}, \ \text{ and } \int_{\text{common}} \vec{B}\cdot d\vec{s}_{\mathrm{I}} = -\int_{\text{common}} \vec{B}\cdot d\vec{s}_{\mathrm{II}}.$$

If we add the line integrals for the two subregions, we get

$$\oint \vec{E}\cdot d\vec{s}_{\mathrm{I}} + \oint \vec{E}\cdot d\vec{s}_{\mathrm{II}} = \int_{\text{whole}} \vec{E}\cdot d\vec{s}_{\mathrm{I}} + \int_{\text{common}} \vec{E}\cdot d\vec{s}_{\mathrm{I}} + \int_{\text{whole}} \vec{E}\cdot d\vec{s}_{\mathrm{II}} + \int_{\text{common}} \vec{E}\cdot d\vec{s}_{\mathrm{II}} = \oint_{\text{whole}} \vec{E}\cdot d\vec{s};$$
$$\oint \vec{B}\cdot d\vec{s}_{\mathrm{I}} + \oint \vec{B}\cdot d\vec{s}_{\mathrm{II}} = \int_{\text{whole}} \vec{B}\cdot d\vec{s}_{\mathrm{I}} + \int_{\text{common}} \vec{B}\cdot d\vec{s}_{\mathrm{I}} + \int_{\text{whole}} \vec{B}\cdot d\vec{s}_{\mathrm{II}} + \int_{\text{common}} \vec{B}\cdot d\vec{s}_{\mathrm{II}} = \oint_{\text{whole}} \vec{B}\cdot d\vec{s};$$

The current is additive, $I = I_{\mathrm{I}} + I_{\mathrm{II}}$, so for every term of Maxwell's third and fourth equations the sum of the terms for the two subregions is the term for the whole region. Thus Maxwell's third and fourth equations are valid for the whole region.

5. From the argument of the cosine function, we see that the wave is traveling in – z-direction. Because $\vec{E}$ and $\vec{B}$ are perpendicular to each other and to the direction of propagation, $\vec{B}$ can have only an x-component, with magnitude $B_0 = E_0/c$:

$$\vec{B} = \boxed{(E_0/c)\cos(kz + \omega t)\,\hat{i}, \quad \text{traveling in the} - z \text{ direction}}.$$

The positive $\hat{i}$-direction is chosen so the wave travels in the – z-direction.

6. For the dimensions, we have

$$[\mu_0\varepsilon_0]^{-1/2} = [\mu_0]^{-1/2}\,[\varepsilon_0]^{-1/2} = [ML/Q^2]^{-1/2}\,[Q^2T^2/ML^3]^{-1/2} = [T^2/L^2]^{-1/2} = [LT^{-1}].$$

7. From the radio dial or the electromagnetic spectrum, FM frequencies are in MHz.

$$f = 94\text{ MHz} = \boxed{94 \times 10^6\text{ Hz}};$$
$$\lambda = c/f = (3 \times 10^8\text{ m/s})/(94 \times 10^6\text{ Hz}) = \boxed{3.2\text{ m}}.$$

8. We replace ε_0 with the permittivity of the medium: $\varepsilon = \kappa\varepsilon_0$:

$$E_0/B_0 = (1/\mu_0\varepsilon)^{1/2} = (1/\kappa\mu_0\varepsilon_0)^{1/2} = \boxed{c/\sqrt{\kappa}}.$$

9. We find the partial second derivatives of the given field:

$E_x = E_0 \sin(kz) \cos(\omega t)$;

$\dfrac{\partial^2 E_x}{\partial t^2} = -E_0\omega^2 \sin(kz)\cos(\omega t)$;

$\dfrac{\partial^2 E_x}{\partial z^2} = -E_0 k^2 \sin(kz)\cos(\omega t) = \dfrac{k^2}{\omega^2}\dfrac{\partial^2 E_x}{\partial t^2} = \dfrac{1}{c^2}\dfrac{\partial^2 E_x}{\partial t^2}$, so $\dfrac{\partial^2 E_x}{\partial z^2} = \mu_0\varepsilon_0\dfrac{\partial^2 E_x}{\partial t^2}$, which is the wave equation.

10. (a) For a frequency of 1000 kHz, we have

$\lambda = c/f = (3\times10^8\ \mathrm{m/s})/(1\times10^6\ \mathrm{Hz}) = \boxed{3\times10^2\ \mathrm{m}}$;

$k = 2\pi/\lambda = 2\pi/(3\times10^2\ \mathrm{m}) = \boxed{2\times10^{-2}\ \mathrm{m}^{-1}}$;

$f = \boxed{1\times10^6\ \mathrm{Hz}}$;

$\omega = 2\pi f = 2\pi(1\times10^6\ \mathrm{Hz}) = \boxed{6\times10^6\ \mathrm{rad/s}}$.

(b) For a frequency of 100 MHz, we have

$\lambda = c/f = (3\times10^8\ \mathrm{m/s})/(1\times10^8\ \mathrm{Hz}) = \boxed{3\mathrm{m}}$;

$k = 2\pi/\lambda = 2\pi/(3\ \mathrm{m}) = \boxed{0.2\ \mathrm{m}^{-1}}$;

$f = \boxed{1\times10^8\ \mathrm{Hz}}$;

$\omega = 2\pi f = 2\pi(1\times10^8\ \mathrm{Hz}) = \boxed{6\times10^8\ \mathrm{rad/s}}$.

(c) For a frequency of 10 GHz, we have

$\lambda = c/f = (3\times10^8\ \mathrm{m/s})/(1\times10^{10}\ \mathrm{Hz}) = \boxed{3\times10^{-2}\ \mathrm{m}}$;

$k = 2\pi/\lambda = 2\pi/(3\times10^{-2}\ \mathrm{m}) = \boxed{2\times10^2\ \mathrm{m}^{-1}}$;

$f = \boxed{1\times10^{10}\ \mathrm{Hz}}$;

$\omega = 2\pi f = 2\pi(1\times10^{10}\ \mathrm{Hz}) = \boxed{6\times10^{10}\ \mathrm{rad/s}}$.

(d) For a wavelength of 600 nm, we have

$\lambda = 600\times10^{-9}\ \mathrm{m} = \boxed{6\times10^{-7}\ \mathrm{m}}$;

$k = 2\pi/\lambda = 2\pi/(6\times10^{-7}\ \mathrm{m}) = \boxed{1\times10^7\ \mathrm{m}^{-1}}$;

$f = c/\lambda = (3\times10^8\ \mathrm{m/s})/(6\times10^{-7}\ \mathrm{m}) = \boxed{5\times10^{14}\ \mathrm{Hz}}$;

$\omega = 2\pi f = 2\pi(5\times10^{14}\ \mathrm{Hz}) = \boxed{3\times10^{15}\ \mathrm{rad/s}}$.

(e) For a wavelength of 0.6 nm, we have

$\lambda = 0.6\times10^{-9}\ \mathrm{m} = \boxed{6\times10^{-10}\ \mathrm{m}}$;

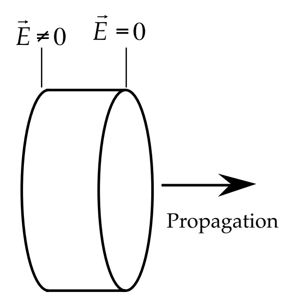
$k = 2\pi/\lambda = 2\pi/(6\times10^{-10}\ \mathrm{m}) = \boxed{1\times10^{10}\ \mathrm{m}^{-1}}$;

$f = c/\lambda = (3\times10^8\ \mathrm{m/s})/(6\times10^{-10}\ \mathrm{m}) = \boxed{5\times10^{17}\ \mathrm{Hz}}$;

$\omega = 2\pi f = 2\pi(5\times10^{17}\ \mathrm{Hz}) = \boxed{3\times10^{18}\ \mathrm{rad/s}}$.

11. We assume that $\vec{E}$ is the same along a wavefront perpendicular to the direction of propagation. We choose a pillbox for a Gaussian surface with the plane surfaces perpendicular to the direction of propagation and one of the plane surfaces at a wavefront where $\vec{E} = \vec{B} = 0$. With no free charge within the pillbox, we have

$\oiint \vec{E}\cdot d\vec{A} = \iint_{\text{planes}} \vec{E}\cdot d\vec{A} + \iint_{\text{side}} \vec{E}\cdot d\vec{A} = 0.$



If $\vec{E}$ at some point on the side is not perpendicular to $d\vec{A}$, there will be a point on the opposite side where $\vec{E}$ is in the same direction but $d\vec{A}$ is in the opposite direction. Thus the surface integral over the side is zero, so we have

$\iint \vec{E}\cdot d\vec{A} = 0$ for the plane surface where $\vec{E} \neq 0$.

This requires that $\vec{E}$ is perpendicular to the plane surface and thus is perpendicular to the direction of propagation.

12. To obtain the wave equation for B_y, we take the partial derivative of Eq. (34–5) with respect to z:

$$\frac{\partial}{\partial z}\left(-\frac{\partial B_y}{\partial z}\right) = \mu_0\varepsilon_0 \frac{\partial}{\partial z}\left(\frac{\partial E_x}{\partial t}\right);$$

$$-\left(\frac{\partial^2 B_y}{\partial z^2}\right) = \mu_0\varepsilon_0\left(\frac{\partial^2 E_x}{\partial z\,\partial t}\right).$$

The partial derivative of Eq. (35–6) with respect to t is

$$\frac{\partial}{\partial t}\left(-\frac{\partial B_y}{\partial t}\right) = \frac{\partial}{\partial t}\left(\frac{\partial E_x}{\partial z}\right);$$

$$-\left(\frac{\partial^2 B_y}{\partial t^2}\right) = \left(\frac{\partial^2 E_x}{\partial t\,\partial z}\right).$$

Because the order of partial differentiation does not matter, we have

$$\left(\frac{\partial^2 B_y}{\partial z^2}\right) = \mu_0\varepsilon_0\left(\frac{\partial^2 B_y}{\partial t^2}\right).$$

This is a wave equation, where the speed is $c = \boxed{(1/\mu_0\varepsilon_0)^{1/2}}$.

13. When we relabel the axes, $x \to y \to z \to x$, we get

$$-\frac{\partial B_z}{\partial x} = \mu_0\varepsilon_0\frac{\partial E_y}{\partial t}, \quad \text{and} \quad -\frac{\partial B_z}{\partial t} = \frac{\partial E_y}{\partial x}.$$

We now have fields which vary in the x-direction. To change the components, we rotate the y- and z-axes 90° around the x-axis, $z \to y \to -z$:

$$\boxed{+\frac{\partial B_y}{\partial x} = \mu_0\varepsilon_0\frac{\partial E_z}{\partial t} \quad \text{and} \quad +\frac{\partial B_y}{\partial t} = \frac{\partial E_z}{\partial x}.}$$

14. (a) For a wave propagating in the $-x$-direction, we have

$\boxed{\vec{E} = E_0\cos(kx + \omega t + \phi_0)\,\hat{k}}$, where $k = 2\pi/\lambda$ and $\omega = 2\pi c/\lambda$.

(b) Using the result from Problem 13, we have

$\partial B_y/\partial t = \partial E_z/\partial x = -kE_0\sin(kx + \omega t + \phi)$;

$\partial B_y/\partial x = \mu_0\varepsilon_0\,\partial E_z/\partial t = -\omega\mu_0\varepsilon_0 E_0\sin(kx + \omega t + \phi)$.

Because $\mu_0\varepsilon_0 = 1/c^2$, these equations are satisfied by

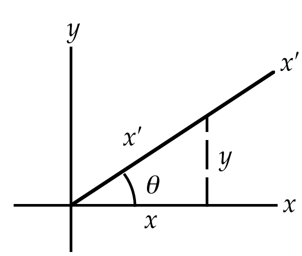
$B_y = (E_0/c)\cos(kx + \omega t + \phi)$.

Because $\vec{E}$ and $\vec{B}$ are perpendicular, $\vec{B}$ will have only a y-component:

$\boxed{\vec{B} = (E_0/c)\cos(kx + \omega t + \phi_0)\,\hat{j}}$.

15. If we choose the direction of propagation as the x'-axis, the electric field is

$\vec{E} = \vec{E}_0\cos(kx' - \omega t + \phi)$.



From the figure, we see that

$x' = x\cos\theta + y\sin\theta$, so we have

$\vec{E} = \vec{E}_0\cos[k(x\cos\theta + y\sin\theta) - \omega t + \phi]$

$= \vec{E}_0\cos(kx\cos\theta + ky\sin\theta - \omega t + \phi)$.

$\vec{E}_0$ can have any direction in the plane that is perpendicular to the direction of propagation, which is $\boxed{\text{the plane formed by the } z\text{-axis and the line } y = -(\tan\theta)x}$.

16. For the plane wave, we have

$$k = 2\pi/\lambda = 2\pi/(17\text{ m}) = 0.37\text{ m}^{-1} \text{ and } \omega = 2\pi c/\lambda = 2\pi(3\times10^8\text{ m/s})/(17\text{ m}) = 1.1\times10^8\text{ rad/s}.$$

The electric field is

$$\vec{E} = E_0 \cos(kz - \omega t + \phi)\hat{j} = \boxed{(0.16\text{ V/m})\cos(0.37z - 1.1\times10^8\ t)\hat{j}}$$, since E is maximum at $z = 0$, $t = 0$.

Here z is in m and t in s.

The amplitude of the magnetic field is

$$B_0 = E_0/c = (0.16\text{ V/m})/(3\times10^8\text{ m/s}) = 5.3\times10^{-10}\text{ T}.$$

To adapt Eq. (35–6), we rotate the coordinate system about the z-axis, $x \to y \to -x$, so we have

$\partial B_x/\partial t = \partial E_y/\partial z = -kE_0 \cos(kz - \omega t)$, which gives

$$\vec{B} = -B_0 \cos(kz - \omega t)\hat{i} = \boxed{-(5.3\times10^{-10}\text{ T})\cos(0.37z - 1.1\times10^8\ t)\hat{i}}$$, with z is in m and t in s.

$\boxed{17}$. For a magnetic field in the y-direction that is maximum at $z = 0$ and $t = 0$, we write

$$\vec{B} = B_0 \cos(kz - \omega t)\hat{j}.$$

The electric field will be perpendicular to $\vec{B}$: $\vec{E} = E_x\hat{i} = \pm E_0 \cos(kz - \omega t)\hat{i}$.

We use Eq. (35–5) to determine if E_x is positive when B_y is positive:

$-\partial B_y/\partial z = \mu_0\varepsilon_0\,\partial E_x/\partial t$;

$+kB_0 \sin(kz - \omega t) = -\mu_0\varepsilon_0(\pm E_0)(-\omega)\sin(kz - \omega t)$, which requires the positive sign.

For the given wavelength we have

$k = 2\pi/\lambda = 2\pi/(600\times10^{-9}\text{ m}) = 1.05\times10^7\text{ m}^{-1}$ and

$\omega = ck = (3\times10^8\text{ m/s})(1.05\times10^7\text{ m}^{-1}) = 3.15\times10^{15}\text{ rad/s}.$

Thus we have

$$\vec{E} = E_0 \cos(kz - \omega t)\hat{i} = cB_0 \cos(kz - \omega t)\hat{i}.$$

$$= (3\times10^8\text{ m/s})(10^{-8}\text{ T})\cos[(1.05\times10^7\text{ m}^{-1})z - (3.15\times10^{15}\text{ rad/s})t]\hat{i}$$

$$= \boxed{(3\text{ V/m})\cos[(1.05\times10^7\text{ m}^{-1})z - (3.15\times10^{15}\text{ rad/s})t]\hat{i}}.$$

18. For the net electric field, we have

$$\vec{E}_{\text{net}} = E_1 \cos(kz - \omega t)\hat{i} + E_1 \cos(kx + \omega t + \phi)\hat{i}.$$

If we use the identity for the sum of two cosines, we get

$$\vec{E}_{\text{net}} = 2E_1 \cos[\tfrac{1}{2}(kz - \omega t + kx + \omega t + \phi)]\cos[\tfrac{1}{2}(kz - \omega t - kx - \omega t - \phi)]\hat{i}$$

$$= 2E_1 \cos(kz + \tfrac{1}{2}\phi)\cos(-\omega t - \tfrac{1}{2}\phi)\hat{i} = 2E_1 \cos(kz + \tfrac{1}{2}\phi)\cos(\omega t + \tfrac{1}{2}\phi)\hat{i},$$

which is the expression for a standing wave.

We find the magnetic field from

$$\partial B_y/\partial t = -\partial E_x/\partial z = +2kE_1 \sin(kz + \tfrac{1}{2}\phi)\cos(\omega t + \tfrac{1}{2}\phi),$$

which, when we integrate, gives

$$\vec{B} = -2(k/\omega)E_1 \sin(kz + \tfrac{1}{2}\phi)\sin(\omega t + \tfrac{1}{2}\phi)\hat{j} = -2(E_1/c)\sin(kz + \tfrac{1}{2}\phi)\sin(\omega t + \tfrac{1}{2}\phi)\hat{j},$$

which is also the expression for a standing wave.

19. The electric field must vanish at $z = 0$ and $z = L$. We see that the given field

$E_x = E_0 \sin(kz)\cos(\omega t) = 0$ at $z = 0$.

At $z = L$ we have

$E_x = E_0 \sin(kL)\cos(\omega t) = 0$, which gives

$\sin(kL) = 0$, or $kL = n\pi$, $n = 1, 2, 3, \ldots$.

The allowed wavelengths are

$$\lambda = \boxed{2\pi/k = 2L/n,\ n = 1, 2, 3, \ldots}.$$

20. A magnetic field would be in the + y-direction for a wave propagating in the + z-direction. Therefore the magnetic field must be in $\boxed{\text{the} - y\text{-direction}}$ for this pulse.
We expect the magnetic field to have the same space-time dependence as the electric field:
$\vec{E} = E_0 e^{-(z+ct)^2/a^2}\,\hat{i},$ and
$\vec{B} = -B_0 e^{-(z+ct)^2/a^2}\,\hat{j}.$
From Eq. (34–5), we have
$$-\frac{\partial B_y}{\partial z} = \mu_0\varepsilon_0 \frac{\partial E_x}{\partial t};$$
$$+[-2(z+ct)/a^2]B_0 e^{-(z+ct)^2/a^2} = \mu_0\varepsilon_0[-2(z+ct)c/a^2]E_0 e^{-(z+ct)^2/a^2}.$$
From Eq. (35–6), we have
$$-\frac{\partial B_y}{\partial t} = \frac{\partial E_x}{\partial z};$$
$$+[-2(z+ct)c/a^2]B_0 e^{-(z+ct)^2/a^2} = [-2(z+ct)/a^2]E_0 e^{-(z+ct)^2/a^2}.$$
Because $\mu_0\varepsilon_0 = 1/c^2$, we see that both of these equations are satisfied if $B_0 = E_0/c$:
$$\boxed{\vec{B} = -(E_0/c)e^{-(z+ct)^2/a^2}\,\hat{j}.}$$

21. We find the amplitude of the magnetic field from
$I = \frac{1}{2}c\varepsilon_0 E_0{}^2 = \frac{1}{2}(c/\mu_0)B_0{}^2;$
$6 \times 10^6\ \text{W/m}^2 = \frac{1}{2}[(3 \times 10^8\ \text{m/s})/(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})]B_0{}^2$, which gives $B_0 = \boxed{2.2 \times 10^{-4}\ \text{T}}$.

22. If the signal is transmitted with equal intensity in all directions, the intensity at a distance R is
$I = P/4\pi R^2 = \frac{1}{2}c\varepsilon_0 E_0{}^2$, which becomes
$E_0{}^2 = 2(1/4\pi\varepsilon_0)P/cR^2.$
At a distance of 3.5 km, we have
$E_0{}^2 = 2(9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(18 \times 10^3\ \text{W})/(3 \times 10^8\ \text{m/s})(3.5 \times 10^3\ \text{m})^2$, which gives $E_0 = \boxed{3.0 \times 10^{-1}\ \text{V/m}}$.
The magnetic field is
$B_0 = E_0/c = (3.0 \times 10^{-1}\ \text{V/m})/(3 \times 10^8\ \text{m/s}) = \boxed{1.0 \times 10^{-9}\ \text{T}}$.
At a distance of 10.5 km, we have
$E_0{}^2 = 2(9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(18 \times 10^3\ \text{W})/(3 \times 10^8\ \text{m/s})(10.5 \times 10^3\ \text{m})^2$, and $E_0 = \boxed{9.9 \times 10^{-2}\ \text{V/m}}$.
The magnetic field is
$B_0 = E_0/c = (9.9 \times 10^{-2}\ \text{V/m})/(3 \times 10^8\ \text{m/s}) = \boxed{3.3 \times 10^{-10}\ \text{T}}$.

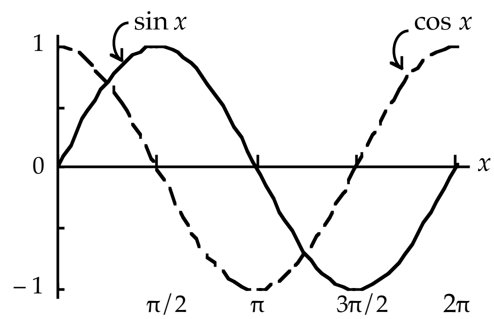
23. We find the intensity from
$I = \frac{1}{2}c\varepsilon_0 E_0{}^2 = \frac{1}{2}(3 \times 10^8\ \text{m/s})(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(140 \times 10^{-3}\ \text{V/m})^2 = \boxed{2.6 \times 10^{-5}\ \text{W/m}^2}$.

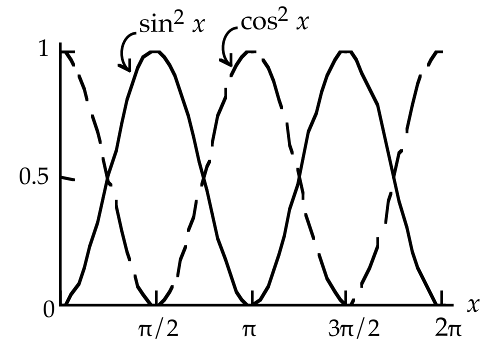
24. Because the light is absorbed, we find the force from
$F = (I/c)A = [(0.40 \times 10^{13}\ \text{W/m}^2)/(3.0 \times 10^8\ \text{m/s})](1.5 \times 10^{-6}\text{m}^2) = \boxed{2.0 \times 10^{-2}\ \text{N}}$.

25. Because the light totally reflects from the surface, the momentum of the beam reverses direction. We find the radiation pressure from
$F/A = 2I/c = 2\langle u\rangle = 2(\frac{1}{2}\varepsilon_0 E_0{}^2) = \varepsilon_0 E_0{}^2$
$= (8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3\ \text{V/m})^2 = \boxed{8 \times 10^{-11}\ \text{N/m}^2}$.

26. Because the light is totally absorbed, the rate of absorption is the rate at which energy is transported to the surface:
$P = IA = c\langle u\rangle A = \frac{1}{2}c\varepsilon_0 E_0{}^2 A.$
The rate per unit area is
$P/A = \frac{1}{2}(3 \times 10^8\ \text{m/s})(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(120\ \text{V/m})^2 = \boxed{38\ \text{W/m}^2}$.

27. (a) We find the magnitude of the Poynting vector from
$$S = cu = I = P/A = (3.8 \times 10^{26}\text{ W})/4\pi(1.5 \times 10^{11}\text{ m})^2 = \boxed{1.3 \times 10^3\text{ W/m}^2}.$$
(b) For a totally absorbing surface, we have
$$F/A = I/c = (1.3 \times 10^3\text{ W/m}^2)/(3 \times 10^8\text{ m/s}) = \boxed{4.5 \times 10^{-6}\text{ N/m}^2}.$$

28. (a) 

(b) 

(c) From the curves we see that $\sin^2 x$ and $\cos^2 x$ are the same curves, shifted by $\frac{1}{2}\pi$. In the interval $0 < x < 2\pi$, both curves have the same number of peaks and thus the same area.
(d) The average of $\sin^2 x$ is the area under the curve divided by Δx. Because both curves have the same area, the averages are the same. We take the average of $\sin^2 x + \cos^2 x = 1$:
$$\langle \sin^2 x\rangle + \langle \cos^2 x\rangle = 2\langle \sin^2 x\rangle = 1, \text{ so } \langle \sin^2 x\rangle = \langle \cos^2 x\rangle = \tfrac{1}{2}.$$
This can also be seen from the curves. The average value is the constant ordinate that will have the same area, which is $\frac{1}{2}$ from the symmetry of the curves around the value $\frac{1}{2}$.

29. We find the intensity from
$$I = \langle S\rangle = (c/\mu_0)\langle B^2\rangle = cB_{\text{rms}}{}^2/\mu_0$$
$$= (3 \times 10^8\text{ m/s})(7 \times 10^{-9}\text{ T})^2/(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A}) = \boxed{1.2 \times 10^{-2}\text{ W/m}^2}.$$
The energy transported in one minute is
$$U = IAt = (1.2 \times 10^{-2}\text{ W/m}^2)(0.1\text{ m}^2)(1\text{ min})(60\text{ s/min}) = \boxed{0.070\text{ J}}.$$

30. (a) We find the peak electric field from
$$I = P/A = \tfrac{1}{2}c\varepsilon_0 E_0{}^2;$$
$$(0.75 \times 10^{-3}\text{ W})/[\tfrac{1}{4}\pi(0.90 \times 10^{-3}\text{ m})^2] = \tfrac{1}{2}(3.0 \times 10^8\text{ m/s})(8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)E_0{}^2,$$
which gives $E_0 = \boxed{9.4 \times 10^2\text{ V/m}}$.
The peak magnetic field is
$$B_0 = E_0/c = (9.42 \times 10^2\text{ V/m})/(3.0 \times 10^8\text{ m/s}) = \boxed{3.1 \times 10^{-6}\text{ T}}.$$
(b) If all of the energy transported in the beam is focused to a circle of diameter λ, we have
$$I_1A_1 = I_2A_2 \quad\text{or}\quad E_{01}{}^2d^2 = E_{02}{}^2\lambda^2; \quad E_{01}d = E_{02}\lambda;$$
$$(9.42 \times 10^2\text{ V/m})(0.90 \times 10^{-3}\text{ m}) = E_{02}(650 \times 10^{-9}\text{ m}), \text{ which gives } E_{02} = \boxed{1.3 \times 10^6\text{ V/m}}.$$

31. If the light radiates with equal intensity in all directions, the intensity at a distance R is
$$I = P/4\pi R^2 = \tfrac{1}{2}c\varepsilon_0 E_0{}^2, \text{ which becomes } E_0{}^2 = (1/4\pi\varepsilon_0)2P/cR^2.$$
At a distance of 0.5 m, we have
$$E_0{}^2 = (9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)2(75\text{ W})/[(3.0 \times 10^8\text{ m/s})(0.50\text{ m})^2], \text{ which gives } E_0 = \boxed{134\text{ V/m}}.$$
For the rms value, we have
$$E_{\text{rms}} = E_0/\sqrt{2} = (134\text{ V/m})/\sqrt{2} = \boxed{95\text{ V/m}}.$$
The magnetic field is
$$B_0 = E_0/c = (134\text{ V/m})/(3.0 \times 10^8\text{ m/s}) = \boxed{4.47 \times 10^{-7}\text{ T}}, \quad\text{and}$$
$$B_{\text{rms}} = B_0/\sqrt{2} = (4.47 \times 10^{-7}\text{ T})/\sqrt{2} = \boxed{3.16 \times 10^{-7}\text{ T}}.$$

32. If the light radiates with equal intensity in all directions, the intensity at a distance R is
$I = P_{\text{visible}}/4\pi R^2 = c\langle u\rangle;$
$0.09(75\text{ W})/[4\pi(1.30\text{ m})^2] = (3.0 \times 10^8\text{ m/s})\langle u\rangle$, which gives $\langle u\rangle = \boxed{1.1 \times 10^{-9}\text{ J/m}^3}$.
We find the rms values of the fields from
$\langle u\rangle = \varepsilon_0 E_{\text{rms}}{}^2 = B_{\text{rms}}{}^2/\mu_0;$
$1.06 \times 10^{-9}\text{ J/m}^3 = (8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)E_{\text{rms}}{}^2 = B_{\text{rms}}{}^2/(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})$, which gives
$E_{\text{rms}} = \boxed{11\text{ V/m}}$, and $B_{\text{rms}} = \boxed{3.6 \times 10^{-8}\text{ T}}$.

33. Because the light totally reflects from the surface, the momentum of the beam reverses direction. We find the radiation pressure from
$F/A = 2\langle u\rangle = 2I/c = 2P/4\pi R^2 c$
$= 2(3.8 \times 10^{26}\text{ W})/4\pi(1.0 \times 10^{10}\text{ m})^2(3.0 \times 10^8\text{ m/s}) = \boxed{2.0 \times 10^{-3}\text{ N/m}^2}$.

34. For the dimensions of the Poynting vector, we have
$[S] = [\mu_0]^{-1}[E][B]$
$= [M^{-1}L^{-1}Q^2][MLQ^{-1}T^{-2}][MQ^{-1}T^{-1}] = \boxed{[MT^{-3}]}$.
The units are kg/s^3, which is also
$\text{kg}\cdot\text{m}^2/\text{s}^3\cdot\text{m}^2 = \text{J/s}\cdot\text{m}^2 = \boxed{\text{W/m}^2}$.

35. With full absorption, the momentum transfer is
$\Delta p = (S/c)A\,\Delta t = (IA/c)\,\Delta t$
$= (160\text{ W/m}^2)(1\text{ m}^2)/(3 \times 10^8\text{ m/s})](365\text{ d})(24\text{ h/d})(3600\text{ s/h}) = \boxed{17\text{ kg}\cdot\text{m/s}}$.
For the momentum change of the baseball, we estimate
$\Delta p = mv \simeq (0.1\text{ kg})(120\text{ km/h})/(3.6\text{ ks/h}) \simeq \boxed{3\text{ kg}\cdot\text{m/s, about 1/5 of the solar result}}$.

36. Because the beam is totally absorbed, the energy density is
$\langle u\rangle = \text{pressure} = 1.0 \times 10^5\text{ N/m}^2 = \boxed{1.0 \times 10^5\text{ J/m}^3}$.
The intensity is
$I = c\langle u\rangle = (3.0 \times 10^8\text{ m/s})(1.0 \times 10^5\text{ J/m}^3) = \boxed{3.0 \times 10^{13}\text{ W/m}^8}$.
We find the rms electric field from
$\langle u\rangle = \varepsilon_0 E_{\text{rms}}{}^2;$
$1.0 \times 10^5\text{ J/m}^3 = (8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)E_{\text{rms}}{}^2$, which gives $E_{\text{rms}} = \boxed{1.1 \times 10^8\text{ V/m}}$.
We find the rms magnetic field from
$B_{\text{rms}} = E_{\text{rms}}/c = (1.06 \times 10^8\text{ V/m})/(3.0 \times 10^8\text{ m/s}) = \boxed{0.35\text{ T}}$.
These values are rather large.

37. To maximize the torque we need to maximize the lever arm, by placing the wire along a shorter edge (h = 20 cm) of the mirror so that the radiation force can be applied along the longer side (L = 500 cm). The mirror is reflective so the optical pressure exerted on it is twice the normal value:
$\text{pressure} = 2(5 \times 10^{-6}\text{ N/m}^2)$.
The force on the mirror is
$F = (\text{pressure})A = PLh.$
Multiply this by the average value of the lever arm, or $\frac{1}{2}L$, to obtain
$\tau = \frac{1}{2}FL = \frac{1}{2}PL^2h = \frac{1}{2}[2(5 \times 10^{-6}\text{ N/m}^2)(5.00\text{ m})^2(0.20\text{ m})] = \boxed{2.5 \times 10^{-5}\text{ N}\cdot\text{m}}$.

38. The radiation pressure one a totally absorbing wall is
$I/c = \frac{1}{2}\varepsilon_0 E_0{}^2 = \frac{1}{2}\varepsilon_0(2E_{\text{rms}}{}^2) = (8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(500\text{ V/m})^2 = 2.2 \times 10^{-6}\text{ N/m}^2.$
If the wall is highly reflective then the pressure on the wall doubles, to $\boxed{4.4 \times 10^{-6}\text{ N/m}^2}$. The actual pressure is somewhat less than this value, since the reflection cannot be 100%.

39. The force exerted by the radiation is

$F_{\text{radiation}} = (\text{pressure})A = uA = (I/c)A = P/c.$

To suspend the paper, the force from the radiation must balance the force of gravity:

$P/c = mg;$

$P/(3.0 \times 10^8\ \text{m/s}) = (0.20 \times 10^{-3}\ \text{kg})(9.8\ \text{m/s}^2)$, which gives $P = \boxed{5.9 \times 10^5\ \text{W}}$.

At this rate of energy absorption, $\boxed{\text{the paper would burn or vaporize}}$.

40. Because the light is totally reflected, the force exerted by the radiation is

$F_{\text{radiation}} = (\text{pressure})A = 2uA = (2I/c)A.$

To suspend the mica, the force from the radiation must balance the force of gravity:

$(2I/c)A = mg;$

$[2I/(3.0 \times 10^8\ \text{m/s})](0.06 \times 10^{-6}\text{m}^2) = (5.4 \times 10^{-9}\ \text{kg})(9.8\ \text{m/s}^2)$, which gives $I = \boxed{1.3 \times 10^8\ \text{W/m}^2}$.

$\boxed{41}$. We consider the beam that falls on an area A of the surface. The cross-sectional area of the beam is $A \cos \theta$. In a time Δt, the volume of the beam that reflects from the surface is $(A \cos \theta)\, c\, \Delta t$.
The momentum in this volume is

$p = (S/c^2)(A \cos \theta)\, c\, \Delta t.$

Upon reflection, only the component of momentum perpendicular to the surface reverses, so the momentum change is

$\Delta p = 2p \cos \theta.$

The momentum transferred to the surface per unit area is

$\Delta p/A = \boxed{2(S/c) \cos^2 \theta\ \Delta t}$.

42. We find the peak electric field from the intensity:

$I = P/A = \Delta U/A\, \Delta t = \tfrac{1}{2}c\varepsilon_0 E_0{}^2;$

$(10^2\ \text{J})/[\tfrac{1}{4}\pi(10^{-4}\ \text{m})^2(10^{-8}\ \text{s})] = \tfrac{1}{2}(3 \times 10^8\ \text{m/s})(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)E_0{}^2$, which gives

$E_0 = \boxed{3.1 \times 10^{10}\ \text{V/m}}$.

The peak magnetic field is

$B_0 = E_0/c = (3.1 \times 10^{10}\ \text{V/m})/(3 \times 10^8\ \text{m/s}) = \boxed{1.0 \times 10^2\ \text{T}}$.

43. Choose the electric field to be polarized in the z-axis and the magnetic field in the y-axis. (Note that $\vec{E} \times \vec{B}$ is along the negative x-direction, or the direction of propagation of the wave, as it should be.) The electric and magnetic fields then have the expressions

$\vec{E}\ (x, t) = E_0 \sin(kx - \omega t)\hat{k}$, $\vec{B}\ (x, t) = B_0 \sin(kx - \omega t)\hat{j}$.

Here $k = \omega/c = 2\pi f/c = 2\pi(6 \times 10^9\ \text{Hz})/(3.0 \times 10^8\ \text{m/s}) = 40\pi\ \text{m}^{-1}$.

To find E_0 and B_0, we note that the optical pressure is

$I/c = \tfrac{1}{2}\varepsilon_0 E_0{}^2$, so

$E_0 = [2(I/c)/\varepsilon_0]^{1/2} = [2(10^{-4}\ \text{N/m}^2)/(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)]^{1/2} = 4.8 \times 10^3\ \text{V/m}$ and

$B_0 = E_0/c = (4.8 \times 10^3\ \text{V/m})/(3.0 \times 10^8\ \text{m/s}) = 1.6 \times 10^{-5}\ \text{T}$. Thus

$\vec{E}\ (x, t) = E_0 \sin(kx - \omega t)\hat{k} = \boxed{(4.8 \times 10^3\ \text{V/m})\sin\{2\pi[(20\ \text{m}^{-1})x - (6 \times 10^9\ \text{Hz})t]\}\hat{k}}$ and

$\vec{B}\ (x, t) = B_0 \sin(kx - \omega t)\hat{j} = \boxed{(1.6 \times 10^{-5}\ \text{T})\sin\{2\pi[(20\ \text{m}^{-1})x - (6 \times 10^9\ \text{Hz})t]\}\hat{j}}$.

44. (a) Since $k = 2\pi/\lambda$, $\lambda = \boxed{2\pi/k}$.

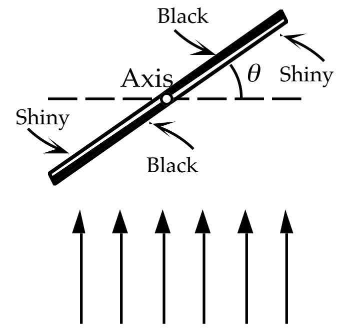
(b) The two component waves making up the standing wave both travel along the x-direction, while the electric field is along the y-direction. Thus $\vec{B}$ must be in the z-axis:

$\vec{B}\ (x, t) = B_0 \sin kx \cos \omega t\ \hat{k} = \boxed{(E_0/c)\sin kx \cos \omega t\ \hat{k}}$.

$\vec{S} = (1/\mu_0)\ \vec{E} \times \vec{B} = (1/\mu_0)(E_0 \sin kx \cos \omega t\, \hat{j}) \times [(E_0/c) \sin kx \cos \omega t\ \hat{k}] = \boxed{(E_0{}^2/\mu_0 c)\sin^2 kx \cos^2 \omega t\ \hat{i}}$.

45. We first consider the situation where the rectangle is perpendicular to the beam. Because the pressure is uniform, the force exerted by the beam on the bright side, where the radiation reflects, is

$$F_{\text{bright}} = 2uA_{\text{bright}} = 2(S/c)H(w/2).$$



The force is uniform over the surface, so the resultant force acts at the center of the bright side, which means a moment arm of $w/4$ about the axis and a torque of

$$\tau_{\text{bright}} = +\,(w/4)(S/c)Hw = +\,SHw^2/4c.$$

The force exerted by the beam on the dark side, where the radiation is absorbed, is

$$F_{\text{dark}} = uA_{\text{dark}} = (S/c)H(w/2).$$

The resultant force acts at the center of the dark side, which means a moment arm of $w/4$ and a torque opposite to that on the bright side of

$$\tau_{\text{dark}} = -\,(w/4)(S/c)H(w/2) = -\,SHw^2/8c.$$

Thus there is a net torque, which is maximum when the surface is perpendicular to the beam:

$$\tau_{\max} = +\,(SHw^2/4c) - (SHw^2/8c) = SHw^2/8c.$$

When the surface has rotated an angle θ, the torque will decrease for two reasons: The area presented by the rectangle to the beam decreases by a factor of $\cos\theta$, and the moment arm decreases by a factor of $\cos\theta$. The torque becomes

$$\tau = \tau_{\max}\cos^2\theta.$$

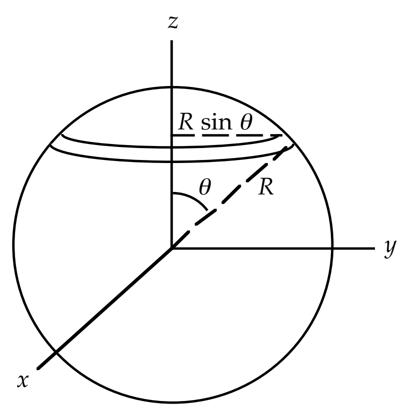
As the other side of the rectangle becomes illuminated, the torque will be in the same direction. The average torque over a full rotation is

$$\tau_{\text{average}} = \tau_{\max}\langle\cos^2\theta\rangle = \tfrac{1}{2}\tau_{\max} = SHw^2/16c.$$

$$= (0.5\ \text{kg/s}^3)(3.0\times10^{-2}\ \text{m})(1.0\times10^{-2}\ \text{m})^2/16(3\times10^8\ \text{m/s}) = \boxed{3.1\times10^{-16}\ \text{N}\cdot\text{m}}.$$

46. The intensity from the dipole will depend on direction and distance, which we can write

$$S = b\sin^2\theta/R^2, \text{ where } b \text{ is a constant.}$$



We determine b from the total power output. Because of the angular dependence, we integrate to find the power through a sphere of radius R centered at the dipole with θ measured from the z-direction, as shown in the figure. Because the intensity pattern is the same above and below the xy-plane, we double the power through the top half of the sphere. We choose a circular strip at angle θ of width $R\,d\theta$ for a differential element, with area $(2\pi R\sin\theta)R\,d\theta$:

$$P = \oiint \vec{S}\cdot d\vec{A} = 2\int_0^{\pi/2}\frac{b\sin^2\theta}{R^2}(2\pi R\sin\theta)R\,d\theta$$

$$= 4\pi b\int_0^{\pi/2}\sin^3\theta\,d\theta = 4\pi b\int_0^{\pi/2}(1-\cos^2\theta)\sin\theta\,d\theta$$

$$= 4\pi b\left(-\cos\theta + \tfrac{1}{3}\cos^3\theta\right)\Big|_0^{\pi/2} = \frac{8\pi b}{3}.$$

Thus we find b:

$$b = 3P/8\pi = 3(20\times10^6\ \text{W})/8\pi = 2.4\times10^6\ \text{W}.$$

The maximum intensity occurs at $\theta = 90°$:

$$S_{\max} = b/R^2 = (2.4\times10^6\ \text{W})/(10^3\ \text{m})^2 = 2.4\ \text{W/m}^2.$$

If the intensity were distributed uniformly in every direction, the intensity would be

$$S = P/4\pi R^2 = (20\times10^6\ \text{W})/4\pi(10^3\ \text{m})^2 = 1.6\ \text{W/m}^2, \text{ so } \boxed{S_{\max} = 1.5\,S_{\text{uniform}}}.$$

47. The frequency of the electromagnetic wave is

$$f = c/\lambda = (3.0 \times 10^8 \text{ m/s})/(120 \text{ m}) = 2.5 \times 10^6 \text{ Hz}.$$

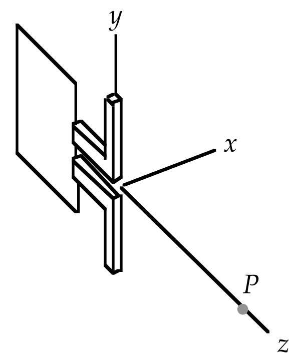
Because the frequency of the current oscillation is the frequency of the wave, we have

$$T = 1/f = 1/(2.5 \times 10^6 \text{ Hz}) = \boxed{4.0 \times 10^{-7} \text{ s}}.$$

48. Because the two observers are the same distance from the charge, for the ratio of intensities we have

$$S_B/S_A = \sin^2\theta_2/\sin^2\theta_1 = (\sin^2 58°)/(\sin^2 25°) = \boxed{4.0}.$$

49. We place the dipole at the origin of the coordinate system.



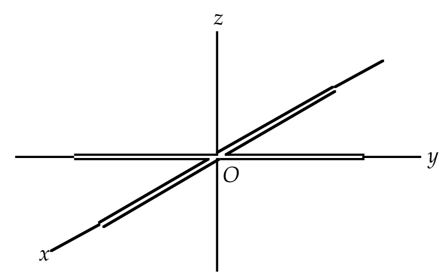
(a) The electric field is parallel to the antenna,
$\boxed{+y\text{-direction}}$.

(b) $\vec{E} \times \vec{B}$ is in the z-direction. The magnetic field is perpendicular to the electric field and the direction of propagation:
$\boxed{-x\text{-direction}}$.

(c) The Poynting vector is in the direction of propagation:
$+z$-direction.

(d) A half-cycle later, the electric and magnetic fields will have reversed, but the direction of propagation will be the same:
electric field in $\boxed{-y\text{-direction}}$,
magnetic field in $\boxed{+x\text{-direction}}$,
Poynting vector in $\boxed{+z\text{-direction}}$.

50. Each arm generates an electric field with amplitude E_0.



(a) Both arms will generate a maximum electric field at the same time. The resultant amplitude will be along the $x = y$ direction, with

$$E_a^2 = E_0^2 + E_0^2 = 2E_0^2.$$

The Poynting vector is

$$\vec{S} = c\varepsilon_0 E^2 \hat{k} = \boxed{2c\varepsilon_0 E^2 \cos^2(kz - \omega t)\,\hat{k}}.$$

(b) Both arms will generate a maximum electric field at the same time. The resultant amplitude will be along the $x = -y$ direction, with

$$E_b^2 = E_0^2 + E_0^2 = 2E_0^2.$$

The Poynting vector is

$$\vec{S} = c\varepsilon_0 E^2 \hat{k} = \boxed{2c\varepsilon_0 E^2 \cos^2(kz - \omega t)\,\hat{k}}.$$

$\boxed{51}$. If the original intensity is I_0, the first Polaroid sheet will reduce the intensity of the original beam to

$$I_1 = \tfrac{1}{2}I_0.$$

If the axis of the second Polaroid sheet is oriented at an angle θ, the intensity is

$$I_2 = I_1\cos^2\theta = \tfrac{1}{2}I_0\cos^2\theta.$$

(a) Because the intensity is already reduced by 1/2, it is not possible to have a final intensity of $(7/10)I_0$, so we have $\boxed{\text{no solution}}$.

(b) For a final intensity of $(3/10)I_0$, we have
$(3/10)I_0 = \tfrac{1}{2}I_0\cos^2\theta$, which gives $\cos^2\theta = 0.6$, $\theta = \boxed{39°}$.

(c) For a final intensity of $(3/20)I_0$, we have
$(3/20)I_0 = \tfrac{1}{2}I_0\cos^2\theta$, which gives $\cos^2\theta = 0.3$, $\theta = \boxed{57°}$.

(d) For a final intensity of $(1/20)I_0$, we have
$(1/20)I_0 = (1/2)I_0\cos^2\theta$, which gives $\cos^2\theta = 0.1$, $\theta = \boxed{72°}$.

52. Through the successive sheets, we have

$I_1 = \frac{1}{2}I_0$,

$I_2 = I_1 \cos^2 \theta$,

$I_3 = I_2 \cos^2 \theta$,

$I_4 = I_3 \cos^2 \theta$, which gives

$I_4/I_0 = \frac{1}{2}\cos^6 \theta = \frac{1}{2}\cos^6 28° = \boxed{0.24}$, or 24%.

53. We find the angle from the vertical from

$\tan \theta_B = n = 1.33$, which gives $\theta_B = 53°$.

The angle above the horizontal is $90° - 53° = \boxed{37°}$.

54. The intensity passing through the sheet is

$I = I_0 \cos^2 \theta, = (1.0 \times 10^6\ \text{W/m}^2) \cos^2 40° = \boxed{5.9 \times 10^5\ \text{W/m}^2}$.

55. The polarizing axis of the second sheet makes an angle of 40° with the polarization direction of the light that passes through the first sheet, so the final intensity is

$I_1 = I \cos^2 \theta, = (5.9 \times 10^5\ \text{W/m}^2) \cos^2 40° = \boxed{3.5 \times 10^5\ \text{W/m}^2}$.

[If the first sheet were not there, the intensity would be $(5.9 \times 10^5\ \text{W/m}^2) \cos^2 80° = 3.0 \times 10^4\ \text{W/m}^2$.]

56. Through the successive sheets, we have

$I_1 = I_0 \cos^2 \theta_1$,

$I_2 = I_1 \cos^2 \theta_2$, which gives

$I_2 = (\cos^2 \theta_1)(\cos^2 \theta_2)I_0 = (\cos^2 33°)(\cos^2 51°)I_0 = \boxed{0.28I_0}$.

$\boxed{57}$. Through the two sheets, we have

$I_1 = \frac{1}{2}I_0$,

$I_2 = I_1 \cos^2 \theta$, which gives

$I_2/I_0 = \frac{1}{2}\cos^2 \theta = \frac{1}{2}\cos^2 90° = \boxed{0}$.

When we insert the third sheet, we have

$I_1 = \frac{1}{2}I_0$,

$I_2 = I_1 \cos^2 \theta_1$,

$I_3 = I_2 \cos^2 \theta_2$, which gives

$I_3/I_0 = \frac{1}{2}(\cos^2 \theta_1)(\cos^2 \theta_2) = \frac{1}{2}(\cos^2 45°)(\cos^2 45°) = \boxed{1/8}$.

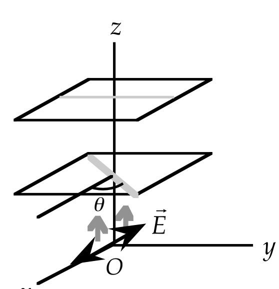
58. Through the successive sheets, we have

$I_1 = I_0 \cos^2 \theta$,

$I_2 = I_1 \cos^2 (90° - \theta)$, which gives

$I_2 = [\cos^2 (90° - \theta)](\cos^2 \theta)I_0$

$= (\sin^2 \theta \cos^2 \theta)I_0 = \boxed{\frac{1}{4}I_0 \sin^2 (2\theta)}$.



59. (a) The first Polaroid sheet will reduce the intensity of the original beam to
$I_1 = \boxed{\tfrac{1}{2}I_0}$.
(b) Through the second sheet, we have
$I_2 = I_1 \cos^2 \theta$.
For the intensity to be zero, we have
$\cos^2 \theta = 0$, which gives $\theta = \boxed{90°}$.
(c) When we insert the third sheet, we have
$I_3 = I_2 \cos^2 (90° - \theta)$, which gives
$I_3 = [\cos^2 (90° - \theta)](\cos^2 \theta)(1/2)I_0 = (1/2)(\sin^2 \theta \cos^2 \theta)I_0 = \boxed{(1/8)I_0 \sin^2 (2\theta)}$.
(d) The intensity will be zero when
$\sin^2(2\theta) = 0$, which gives $2\theta = 0°$ or $180°$, and thus $\theta = 0°$ or $90°$.

60. The angle θ between the polarization direction and the axis of the polarizer satisfies
$I/I_0 = \cos^2\theta = 3/4;$ so $\theta = 30°$.
We must then $\boxed{\text{rotate the polarizer back by } 30°}$ to align its axis with the direction of polarization of the wave to allow for 100% transmission.

61. The electric field vector is given by
$\vec{E}(z, t) = E_0 (\sin\theta\, \hat{i} + \cos\theta\, \hat{j})$, where $\theta = kz - \omega t$.
The direction of the field is along the unit vector $(\sin\theta\, \hat{i} + \cos\theta\, \hat{j})$, which lies in the xy plane and makes an angle with the + x-direction. For a given location (fixed z), as time progresses, $\theta = kz - \omega t$ changes at an angular frequency ω, so the unit vector rotates in the xy plane (i.e., about the z-axis) at an angular speed ω.
Since the Maxwell's equations are linear in the absence of charges, the linear superposition of any of their two solutions is also a solution. So the circularly polarized wave satisfies the Maxwell's equations and is therefore physically acceptable.

62. We equate the energies:
$N h f_{\text{radio}} = hf_{\text{light}} = hc/\lambda_{\text{light}};$
$N(4.5 \times 10^8 \text{ Hz}) = (3.0 \times 10^8 \text{ m/s})/(450 \times 10^{-9} \text{ m})$, which gives $N = \boxed{1.5 \times 10^6 \text{ photons}}$.

63. The frequency and energy of a photon at this wavelength are
$f = c/\lambda = (3 \times 10^8 \text{ m/s})/(630 \times 10^{-9} \text{ m}) = 4.8 \times 10^{14} \text{ Hz};$
$hf = (6.63 \times 10^{-34} \text{ J}\cdot\text{s})(4.8 \times 10^{14} \text{ Hz}) = 3.2 \times 10^{-19} \text{ J}.$
The time for a photon to travel through the length of the apparatus is $\Delta t = L/c$. If we want one photon to be present in the apparatus, the power of the beam is
$P = hf/\Delta t = hfc/L,$ and the intensity is
$I = \langle S \rangle = P/A = hfc/LA$
$= (3.2 \times 10^{-19} \text{ J})(3 \times 10^8 \text{ m/s})/(2 \text{ m})(1 \times 10^{-6} \text{m}^2) = \boxed{4.7 \times 10^{-5} \text{ W/m}^2}$.

64. We find the number of photons emitted per second from
$P = (\mathrm{d}N/\mathrm{d}t)hf = (\mathrm{d}N/\mathrm{d}t)hc/\lambda;$
$3.8 \times 10^{26} \text{ W} = (\mathrm{d}N/\mathrm{d}t)(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m/s})/(550 \times 10^{-9} \text{ m})$, which gives
$\mathrm{d}N/\mathrm{d}t = \boxed{1.05 \times 10^{45} \text{ photons/s}}$.

65. We choose the axis of the solenoid as the z-axis.

(a) The magnitude of the magnetic field is

$B = \mu_0 nI = \mu_0 nI_0 \cos(\omega t)$, so we have

$\vec{B} = \boxed{\mu_0 nI \cos \omega t\ \hat{k} \text{ (along the axis)}}$.

(b) From the cylindrical symmetry, the electric field will be circular. We apply Faraday's law to a circular path with radius $r < R$:

$\oint \vec{E}\cdot d\vec{s} = -(d/dt)\int \vec{B}\cdot d\vec{A}$;

$E2\pi r = -\pi r^2 \mu_0 nI_0\,(d/dt)\cos(\omega t) = \pi r^2 \mu_0 nI_0 \omega \sin(\omega t)$, which gives

$\vec{E} = \boxed{\tfrac{1}{2}\mu_0 nI_0 \omega r \sin(\omega t) \text{ (circular)}}$.

(c) We find the Poynting vector from

$\vec{S} = (1/\mu_0)\ \vec{E}\times\vec{B} = (1/\mu_0)EB\hat{r} = \tfrac{1}{2}\mu_0 n^2 I_0^{\,2}\omega r \sin(\omega t)\cos(\omega t)\hat{r}$;

$\vec{S} = (1/4)\mu_0 n^2 I_0^{\,2}\omega r \sin(2\omega t)\hat{r}$.

The period of oscillation of the fields is $T = 2\pi/\omega$. For each quarter-cycle, we have

$0 < t < \tfrac{1}{4}T$: $\vec{S}$ is in $\boxed{+\hat{r}}$

$\tfrac{1}{4}T < t < \tfrac{1}{2}T$: $\vec{S}$ is in $\boxed{-\hat{r}}$;

$\tfrac{1}{2}T < t < \tfrac{3}{4}T$: $\vec{S}$ is in $\boxed{+\hat{r}}$;

$\tfrac{3}{4}T < t < T$: $\vec{S}$ is in $\boxed{-\hat{r}}$.

66. The force on a charge is

$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$.

For the electromagnetic wave, the two fields are related by $E = cB$. If we assume that $\vec{v}$ and $\vec{B}$ are perpendicular, the ratio of the magnitudes of the forces is

$F_B/F_E = vB/E = v/c = 0.30$, which gives $v = 0.30(3.0\times 10^8\ \text{m/s}) = \boxed{9.0\times 10^7\ \text{m/s}}$.

For a wave traveling in the z-direction, with E in the x-direction, the magnetic field will be in the y-direction. The magnetic force is greatest when the velocity is perpendicular to the magnetic field, so the velocity will be in the $\boxed{xz\text{-plane}}$.

67. We assume that the radiation totally reflects from the sail. The radiation force will be away from the sun, with magnitude

$F_{\text{radiation}} = 2uA = 2IA/c$.

Because the energy through a sphere of radius r must be independent of r, we have

$I4\pi r^2 = k'$.

The gravitational force is toward the sun, so the net force is

$F_{\text{net}} = F_{\text{radiation}} - F_{\text{grav}}$

$= 2k'A/4\pi cr^2 - GMm/r^2 = k/r^2$, where $k = k'A/2\pi c - GMm$.

The value of k will not change, so the force will always have the same sign.

68. (a) We assume that the radiation totally reflects from the sail. The radiation pressure will be directed away from the sun with magnitude

$P = 2u = 2I/c$.

Because the sail is at the radius of the earth's orbit, the pressure is

$P = 2(1.4\times 10^3\ \text{W/m}^2)/(3\times 10^8\ \text{m/s}) = \boxed{9.3\times 10^{-6}\ \text{N/m}^2}$.

(b) We let A be the area of the sail and ℓ be the thickness. If the radiation pressure cancels the gravitational attraction, we have

$PA = GMm/r^2 = GM\rho A\ell/r^2$, or

$\ell = Pr^2/GM\rho$;

$\ell = (9.3\times 10^{-6}\ \text{N/m}^2)(1.5\times 10^{11}\ \text{m})^2/(6.67\times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(2.0\times 10^{30}\ \text{kg})(2\times 10^3\ \text{kg/m}^3)$

$= \boxed{7.9\times 10^{-7}\ \text{m}}$.

Note that the result from Problem 61 shows that the two forces will cancel at any value of r.

69. (a) We find the angular frequency of the wave:

$\omega = 2\pi f = 2\pi \times 10^{14}$ rad/s.

(b) We find the speed of the wave:

$v = c/n = (3 \times 10^8 \text{ m/s})/1.4 = 2.14 \times 10^8$ m/s.

The wave number is

$k = \omega/v = (2\pi \times 10^{14} \text{ rad/s})/(2.14 \times 10^8 \text{ m/s}) = 2.93 \times 10^6 \text{ m}^{-1}$.

(c) If x' is the direction of propagation, we have

$x' = x \cos 30° + y \sin 30°$.

(d) For a wave polarized along the z-axis, we have

$\vec{E} = E\hat{k}$.

(e) For a dielectric medium the wave travels at a speed $v = c/n$, so $B = E/v = nE/c$. We find the amplitude of the electric field from the magnitude of the Poynting vector:

$S = (1/\mu_0)EB = (n/\mu_0 c)E^2 = \frac{1}{2}nc\varepsilon_0 E_0{}^2$;

$500 \text{ W/m}^2 = \frac{1}{2}(1.4)(3 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)E_0{}^2$, which gives $E_0 = 5.19 \times 10^2$ V/m.

The electric field is

$$\vec{E} = E_0 \cos(kx' - \omega t)\hat{k} = E_0 \cos[k(x \cos 30° + y \sin 30°) - \omega t]\hat{k}$$

$$= (5.19 \times 10^2 \text{ V/m}) \cos[(2.93 \times 10^6 \text{ m}^{-1}) \cos 30°\, x + (2.93 \times 10^6 \text{ m}^{-1}) \sin 30°\, y - (2\pi \times 10^{14} \text{ s}^{-1})t]\hat{k}$$

$$= \boxed{(5.19 \times 10^2 \text{ V/m})\cos[(2.54 \times 10^6 \text{ m}^{-1})x - (1.47 \times 10^6 \text{ m}^{-1})y - (2\pi \times 10^{14} \text{s}^{-1})t]\hat{k}}.$$

70. Because the light is coming from water to air, we find the angle from the vertical from

$\tan \theta_B = 1/n_{\text{water}} = 1/1.33$, which gives $\theta_B = \boxed{37°}$.

The angle below the horizontal is $90° - 37° = 53°$.

71. We find the energy absorbed from

$U = (\text{fraction})IAt$

$= (0.40)(800 \text{ W/m}^2)(0.5 \text{ m}^2)(1 \text{ h})(3600 \text{ s/h}) = \boxed{5.8 \times 10^5 \text{ J}}$.

Most of the dissipated energy evaporates the perspiration, so we have

$U = mL_v$;

$5.8 \times 10^5 \text{ J} = m(41 \times 10^3 \text{ J/mol})/(18 \text{ g/mol})$, which gives

$m = 250 \text{ g} = \boxed{0.25 \text{ kg}}$ $(\simeq \frac{1}{4}$ liter).

72. The intensity of the focused beam is

$I = P/A = (15 \times 10^6 \text{ W})/[(0.60 \text{ mm}^2)(10^{-3} \text{ m/mm})^2] = \boxed{2.5 \times 10^{13} \text{ W/m}^2}$.

We find the peak electric field from

$I = \frac{1}{2}c\varepsilon_0 E_0{}^2$;

$2.5 \times 10^{13} \text{ W/m}^2 = \frac{1}{2}(3.0 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)E_0{}^2$, which gives $E_0 = \boxed{1.4 \times 10^8 \text{ V/m}}$.

The peak magnetic field is

$B_0 = E_0/c = (1.37 \times 10^7 \text{ V/m})/(3.0 \times 10^8 \text{ m/s}) = \boxed{0.46 \text{ T}}$.

We find the average energy density from

$I = c\langle u\rangle$;

$2.5 \times 10^{13} \text{ W/m}^2 = (3.0 \times 10^8 \text{ m/s})\langle u\rangle$, which gives $\langle u\rangle = \boxed{8.3 \times 10^4 \text{ J/m}^3}$.

The electric field is greater than the breakdown electric field in air.
The magnetic field is the value of the magnetic field near the pole of a magnet.

73. We find the number of photons/m^3 from the energy density of the beam:

$\langle u\rangle = \frac{1}{2}\varepsilon_0 E_0{}^2 = nhf = nhc/\lambda$;

$\frac{1}{2}(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(10 \text{ V/m})^2 = n(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m/s})/(2 \times 10^{-2} \text{ m})$, which gives

$n = \boxed{4.5 \times 10^{13} \text{ photons/m}^3}$.

74. (a) The energy flux is the same over the surface of the sphere centered on the sun, so we have

$$P = IA = I4\pi R_0^2 = (1.4 \times 10^3\ \mathrm{W/m^2})4\pi(1.5 \times 10^{11}\ \mathrm{m})^2 = \boxed{4.0 \times 10^{26}\ \mathrm{W}}.$$

(b) We find the rate at which photons are emitted from

$$P = (\mathrm{d}N/\mathrm{d}t)hf = (\mathrm{d}N/\mathrm{d}t)hc/\lambda;$$

$$4.0 \times 10^{26}\ \mathrm{W} = (\mathrm{d}N/\mathrm{d}t)(6.63 \times 10^{-34}\ \mathrm{J \cdot s})(3 \times 10^8\ \mathrm{m/s})/(600 \times 10^{-9}\ \mathrm{m}),$$ which gives

$$\mathrm{d}N/\mathrm{d}t = \boxed{1.2 \times 10^{45}\ \text{photons/s}}.$$

(c) Because the intensity of the photons is the same over the surface of the sphere, we have

$$(\mathrm{d}N'/\mathrm{d}t)/A = (\mathrm{d}N/\mathrm{d}t)/4\pi R_0^2;$$

$$(\mathrm{d}N'/\mathrm{d}t)/(1\ \mathrm{mm^2})(10^{-3}\ \mathrm{m/mm})^2 = (1.2 \times 10^{45}\ \text{photons/s})/4\pi(1.5 \times 10^{11}\ \mathrm{m})^2,$$ which gives

$$\mathrm{d}N'/\mathrm{d}t = \boxed{4.2 \times 10^{15}\ \text{photons/s}}.$$

75. We let M be the mass of the mirror, v' be the final speed of the mirror, and f' be the frequency of the reflected photons. For energy conservation, we have

$$Nhf + \tfrac{1}{2}Mv^2 = Nhf' + \tfrac{1}{2}Mv'^2,$$ which becomes

$$v' = v[1 + 2Nh(f - f')/Mv^2]^{1/2}.$$

Because the photon energies are much smaller than the mirror kinetic energy, we get

$$v' \simeq v\{1 + \tfrac{1}{2}[2Nh(f - f')/Mv^2]\} = v + Nh(f - f')/Mv.$$

For momentum conservation, we have

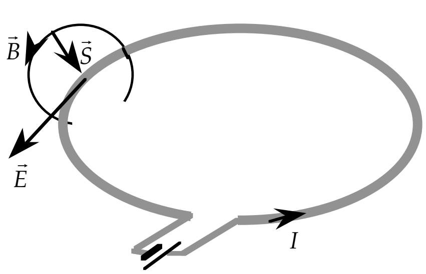
$$Nhf/c + Mv = -(Nhf'/c) + Mv'.$$

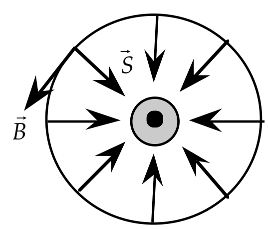
If we substitute the result from energy conservation, we get

$$Nhf/c + Mv = -Nhf'/c + Mv + Nh(f - f')/Mv,$$ which becomes

$$f/c = -f'/c + f/v - f'/v,$$ which gives $f' = \boxed{[(c - v)/(c + v)]f}$.

76. The electric field is along the direction of the current, while the magnetic field follows the right-hand-rule. The resulting Poynting vector points radially inward, towards the wire, as shown. In the figure on the right below, the current flow (and the electric field) is directly out of the paper, and the Poynting vector points radially toward the wire.





77. The distance between Jupiter and the Sun is about 5.2 times the Earth-Sun distance, so the intensity of sunlight at Jupiter's orbit is

$$I_J = I_E(1/5.2)^2 = (1400\ \mathrm{W/m^2})(1/5.2)^2 = 51.8\ \mathrm{W/m^2}.$$

The corresponding optical pressure is I_J/c, and for a highly reflective sail of area A (facing the Sun) the total force due to the optical pressure is

$$F_J = 2(I_J/c)A = ma,$$ which gives the acceleration a_J of the sail as

$$a_J = 2AI_J/mc = 2(100\ \mathrm{m^2})(51.8\ \mathrm{W/m^2})/[(150\ \mathrm{kg})(3.0 \times 10^8\ \mathrm{m/s})] = \boxed{2.3 \times 10^{-7}\ \mathrm{m/s^2}}.$$

The acceleration above is proportional to I, which is in turn inversely proportional to the squared of the distance to the Sun. At the orbit of Saturn, therefore, the acceleration of the sail is

$$a_S = a_J(R_J/R_S)^2 = (2.3 \times 10^{-7}\ \mathrm{m/s^2})(1/1.8)^2 = \boxed{7.1 \times 10^{-8}\ \mathrm{m/s^2}}.$$

As an approximation, we take the average acceleration

$a_{\mathrm{av}} = \tfrac{1}{2}(a_J + a_S) = \tfrac{1}{2}(2.3 \times 10^{-7}\ \mathrm{m/s^2} + 7.1 \times 10^{-8}\ \mathrm{m/s^2}) = 1.5 \times 10^{-7}\ \mathrm{m/s^2}$ and find the time t it takes to sail from Jupiter's to Saturn's orbit, assuming that $a = a_{\mathrm{av}}$:

$$d \approx \tfrac{1}{2}at^2, \text{ or } \quad t \approx (2d/a)^{1/2} = [2(6.5 \times 10^{11}\ \mathrm{m})/(1.5 \times 10^{-7}\ \mathrm{m/s^2})]^{1/2} = 2.9 \times 10^9\ \mathrm{s} = \boxed{93\ \text{years}}.$$

78. We choose the direction of current as the z-axis. We find the electric field inside and on the surface from the current density:

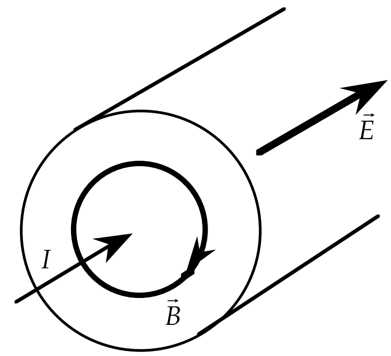
$\vec{E} = \vec{J}/\sigma = (I/A\sigma)\hat{k} = \boxed{(IR/L)\hat{k}}$.

From the cylindrical symmetry, we know that the magnetic field will be circular, centered on the axis of the wire. We choose a circular path with radius $r < R$ to apply Ampere's law:

$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{enclosed}}$;

$B2\pi r = \mu_0 J\pi r^2 = \mu_0 I r^2/R^2$, which gives

$\vec{B} = \boxed{\mu_0 I r/2\pi R^2 \text{ circular, for } r < R}$.



The Poynting vector on the surface of the wire at $r = R$,

$\vec{S} = (1/\mu_0)\,\vec{E}\times\vec{B}$,

will be directed toward the axis of the wire:

$\vec{S} = -(1/\mu_0)(IR/L)(\mu_0 I/2\pi R)\hat{r} = \boxed{-(I^2R/2\pi RL)\hat{r}}$.

The energy flow into the wire is

$P = S(\text{surface area}) = (I^2R/2\pi RL)(2\pi RL) = I^2R$.

79. With the extra term $E_x/\varepsilon_0\rho$ following $\partial E_x(z, t)/\partial t$, Eq. (34-5) becomes

$$-\frac{\partial B_y(z,t)}{\partial z} = \mu_0\varepsilon_0\left[\frac{\partial E_x(z,t)}{\partial t} + \frac{1}{\varepsilon_0\rho}E_x(z,t)\right] = \mu_0\varepsilon_0\frac{\partial E_x(z,t)}{\partial t} + \frac{\mu_0}{\rho}E_x(z,t).$$

Differentiate each term with respect to t to obtain

$$-\frac{\partial^2 B_y(z,t)}{\partial z\partial t} = \mu_0\varepsilon_0\frac{\partial^2 E_x(z,t)}{\partial t^2} + \frac{\mu_0}{\rho}\frac{\partial E_x(z,t)}{\partial t}.$$

Now differentiate both sides of Eq. (34-6) with respect to z :

$$-\frac{\partial^2 B_y(z,t)}{\partial z\partial t} = \frac{\partial^2 E_x(z,t)}{\partial z^2}.$$

Equate the two expressions for $-\partial^2 B_y(z, t)/\partial z\partial t$ above to obtain the modified wave equation for the electric field:

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} = \varepsilon_0\mu_0\frac{\partial^2 E_x}{\partial t^2} + \frac{\mu_0}{\rho}\frac{\partial E_x}{\partial t}}.$$

80. We find the energy of the electromagnetic wave from the energy density:

$U = \langle u\rangle L^3 = \varepsilon_0\langle E^2\rangle L^3 = \varepsilon_0 E_0^2\langle\sin^2(kz - \omega t)\rangle L^3 = \frac{1}{2}\varepsilon_0 E_0^2 L^3$.

The energy of the N photons in the box is

$U = Nhf$.

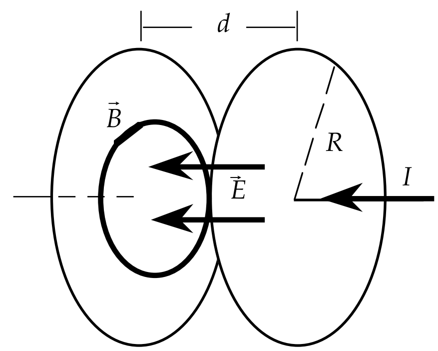
If we equate these two expressions, we have

$\frac{1}{2}\varepsilon_0 E_0^2 L^3 = Nhf$, which gives

$E_0 = \boxed{(2Nhf/\varepsilon_0 L^3)^{1/2}}$.

81. We choose the axis of the circular plates as the z-axis.
We find the electric field from the charge density:

$$\vec{E} = (\sigma/\varepsilon_0)\,\hat{k} = \boxed{(Q/\pi R^2\varepsilon_0)\,\hat{k}}.$$



(a) From the cylindrical symmetry, we know that the magnetic field will be circular, centered on the z-axis. We choose a circular path with radius $r < R$ to apply Ampere's law:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{enclosed}} + \mu_0\varepsilon_0\,(d/dt)\oint \vec{E}\cdot d\vec{A};$$

$$B2\pi r = 0 + \mu_0\varepsilon_0\,(d/dt)[(Q/\pi R^2\varepsilon_0)\pi r^2], \text{ which gives}$$

$$\vec{B} = \boxed{(\mu_0 r/2\pi R^2)\,dQ/dt \text{ circular}}, \text{ for } r < R.$$

(b) The Poynting vector,

$$\vec{S} = (1/\mu_0)\,\vec{E}\times\vec{B},$$

will be directed toward the axis of the plates:

$$\vec{S} = -(1/\mu_0)(Q/\pi R^2\varepsilon_0)(\mu_0 r/2\pi R^2)(dQ/dt)\,\hat{r}$$

$$= \boxed{-(Qr/2\pi^2R^4\varepsilon_0)(dQ/dt)\hat{r}}.$$

(c) The energy flow into the capacitor through the cylinder at $r = R$ is

$$P = S(\text{surface area}) = (Qr/2\pi^2\varepsilon_0R^4)(dQ/dt)(2\pi Rd)$$

$$= [Q/(\varepsilon_0\pi R^2/d)]\,dQ/dt = (d/dt)(Q^2/2C),$$

which is the rate of change of the capacitor energy.